EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Fachbereich Informatik

Programmiersprachen und Softwaretechnik

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Responsible for the lab
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Programming Languages 2

Homework 11 – WS 18

Tübingen, 17. Januar 2019

In order to be admitted to the exam, you have to successfully submit your homework every week, except for 2 weeks. A successful submission is one where you get at least 1 point.

Handin Please submit this homework until Thursday, January 24, either via email to Philipp Schuster (philipp.schuster@uni-tuebingen.de) before 12:00, or on paper at the beginning of the lab.

Groups You can work in groups of up to 2 people. Please include the names and Matrikelnummern of all group members in your submission.

Points For each of the Tasks you get between 0 and 2 points for a total of 6 points. You get:
1 point, if your submission shows that you tried to solve the task.
2 points, if your submission is mostly correct.

Task 1: Natural deduction

Consider these rules of natural deduction:

$$\frac{}{\Gamma, A \vdash A}\,Ax \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \wedge B}\,\wedge_I \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A}\,\wedge_{E1} \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B}\,\wedge_{E2} \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B}\,\Rightarrow_I \qquad \frac{\Gamma \vdash A \Rightarrow B \qquad \Gamma \vdash A}{\Gamma \vdash B}\,\Rightarrow_E$$

Using these rules, show that $((A \Rightarrow B) \wedge (B \Rightarrow C)) \Rightarrow (A \Rightarrow C)$ is derivable.

Task 2: Programs are proofs

Construct a term t in System F (extended with pairs), that has type $((A \to B) \times (B \to C)) \to A \to C$ in context $\Gamma = \{A, B, C\}$. Prove that your term has this type by drawing a derivation tree.

Task 3: Law of excluded middle

Show that the law of excluded middle follows from double negation elimination. Construct a term in System F (extended with sum types) that has type $\forall A.\ A + (\forall Z.\ A \to Z)$. Assume a context $\Gamma = \{\mathrm{dne} : \forall A.\ (\forall X.\ (\forall Y.\ A \to Y) \to X) \to A\}$. No derivation tree necessary.