In order to be admitted to the exam, you have to successfully submit your homework every week, except for 2 weeks. A successful submission is one where you get at least 1 point.

**Handin** Please submit this homework until Thursday, January 24, either via email to Philipp Schuster (philipp.schuster@uni-tuebingen.de) before 12:00, or on paper at the beginning of the lab.

**Groups** You can work in groups of up to 2 people. Please include the names and Matrikelnummern of all group members in your submission.

**Points** For each of the Tasks you get between 0 and 2 points for a total of 6 points. You get:

- 1 point, if your submission shows that you tried to solve the task.
- 2 points, if your submission is mostly correct.

**Task 1: Natural deduction**

Consider these rules of natural deduction:

\[
\begin{array}{|c|c|}
\hline
\text{Ax} & \begin{array}{c}
\text{Γ} \vdash A
\end{array} \\
\hline
\text{Γ}, A \vdash A
\end{array},
\begin{array}{|c|c|}
\hline
\land I & \begin{array}{c}
\text{Γ} \vdash A \\
\text{Γ} \vdash B
\end{array} \\
\hline
\text{Γ} \vdash A \land B
\end{array},
\begin{array}{|c|c|}
\hline
\land E_1 & \begin{array}{c}
\text{Γ} \vdash A \land B
\end{array} \\
\hline
\text{Γ} \vdash A
\end{array},
\begin{array}{|c|c|}
\hline
\land E_2 & \begin{array}{c}
\text{Γ} \vdash A \land B
\end{array} \\
\hline
\text{Γ} \vdash B
\end{array},
\begin{array}{|c|c|}
\hline
\Rightarrow I & \begin{array}{c}
\text{Γ}, A \vdash B
\end{array} \\
\hline
\text{Γ} \vdash A \Rightarrow B
\end{array},
\begin{array}{|c|c|}
\hline
\Rightarrow E & \begin{array}{c}
\text{Γ} \vdash A \Rightarrow B
\end{array} \\
\hline
\text{Γ} \vdash A
\end{array}
\end{array}
\]

Using these rules, show that \(((A \Rightarrow B) \land (B \Rightarrow C)) \Rightarrow (A \Rightarrow C)\) is derivable.

**Task 2: Programs are proofs**

Construct a term \( t \) in System \( F \) (extended with pairs), that has type \(((A \rightarrow B) \times (B \rightarrow C)) \rightarrow A \rightarrow C\) in context \( \Gamma = \{A, B, C\} \). Prove that your term has this type by drawing a derivation tree.
Task 3: Law of excluded middle

Show that the law of excluded middle follows from double negation elimination. Construct a term in System F (extended with sum types) that has type $\forall A. A + (\forall Z. A \rightarrow Z)$. Assume a context $\Gamma = \{\text{dne} : \forall A. (\forall X. (\forall Y. A \rightarrow Y) \rightarrow X) \rightarrow A\}$. No derivation tree necessary.