**Programming Languages 2**

Homework 6 – WS 18

Tübingen, 29. November 2018

In order to be admitted to the exam, you have to successfully submit your homework every week, except for 2 weeks. A successful submission is one where you get at least 1 point.

Handin  Please submit this homework until Thursday, December 06, either via email to Philipp Schuster (philipp.schuster@uni-tuebingen.de) before 12:00, or on paper at the beginning of the lab.

Groups  You can work in groups of up to 2 people. Please include the names and Matrikelnummern of all group members in your submission.

Points  For each of the Tasks you get between 0 and 2 points for a total of 6 points. You get:
   1 point, if your submission shows that you tried to solve the task.
   2 points, if your submission is mostly correct.

**Task 1: Simply typed lambda calculus**

We consider the simply typed lambda calculus from the lecture, extended with unit and let. Show that the following terms are well typed in the given contexts by drawing a derivation tree for the typing relation:

1. \( y : T \vdash (\lambda x : T . x) y : T \)
2. \( \vdash \text{let } f = (\lambda u : \text{Unit} . u) \text{ in } (\lambda x : \text{Unit} . f \text{ unit}) : \text{Unit} \)

**Task 2: Pairs, Tuples, and Records**

We consider the simply typed lambda calculus with all extensions presented in the lecture. For which of the following terms \( t \) does a context \( \Gamma \) and a type \( T \) exist, such that they are well typed. In other words \( \Gamma \vdash t : T \)? If they exist, please write down \( \Gamma \) and \( T \). If not, a short note is enough.

1. \( \lambda b : \text{Bool} . \text{if } b \text{ then(iszero } p.1 \text{) else(} p.2 \text{)} \)
2. \( x.4 \)
3. \( \text{iszero(r. age)} \)
Task 3: Substitution Lemma

We extend the simply typed lambda calculus with false, true and if $t_0$ then $t_1$ else $t_2$ with typing rules from the lecture. We extend the definition of substitution by the following three cases:

... 

\[
\begin{align*}
[x \mapsto s] \text{false} &= \text{false} \\
[x \mapsto s] \text{true} &= \text{true} \\
[x \mapsto s]\text{if } t_0 \text{ then } t_1 \text{ else } t_2 &= \text{if } [x \mapsto s]t_0 \text{ then } [x \mapsto s]t_1 \text{ else } [x \mapsto s]t_2
\end{align*}
\]

Show that if $\Gamma, x : S \vdash t : T$ and $\Gamma \vdash s : S$ then $\Gamma \vdash [x \mapsto s]t : T$. Hint: try induction on the typing derivation $\Gamma, x : S \vdash t : T$. 

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