EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Fachbereich Informatik

Programmiersprachen und Softwaretechnik

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Programming Languages 2

Homework 6 – WS 18

Tübingen, 29. November 2018

In order to be admitted to the exam, you have to successfully submit your homework every week, except for 2 weeks. A successful submission is one where you get at least 1 point.

Handin Please submit this homework until Thursday, December 06, either via email to Philipp Schuster (philipp.schuster@uni-tuebingen.de) before 12:00, or on paper at the beginning of the lab.

Groups You can work in groups of up to 2 people. Please include the names and Matrikelnummern of all group members in your submission.

Points For each of the Tasks you get between 0 and 2 points for a total of 6 points. You get:
1 point, if your submission shows that you tried to solve the task.
2 points, if your submission is mostly correct.

Task 1: Simply typed lambda calculus

We consider the simply typed lambda calculus from the lecture, extended with unit and let. Show that the following terms are well typed in the given contexts by drawing a derivation tree for the typing relation:

1. $y : T \vdash (\lambda x : T.\, x)\; y \quad : \quad T$
2. $\vdash \mathrm{let}\, f = (\lambda u : \mathrm{Unit}\,.\; u)\, \mathrm{in}\, (\lambda x : \mathrm{Unit}\,.\; f \;\; \mathrm{unit}) \quad : \quad \mathrm{Unit}$

Task 2: Pairs, Tuples, and Records

We consider the simply typed lambda calculus with all extensions presented in the lecture. For which of the following terms t does a context Γ and a type T exist, such that they are well typed. In other words $\Gamma \vdash t : T$? If they exist, please write down Γ and T. If not, a short note is enough.

1. $\lambda b : \mathrm{Bool}\,.\;\; \mathrm{if}\, b\, \mathrm{then}(\mathrm{iszero}\, p.1)\, \mathrm{else}(p.2)$
2. $x.4$
3. $\mathrm{iszero}(r.\, \mathrm{age})$

Task 3: Substitution Lemma

We extend the simply typed lambda calculus with false, true and $\text{if}\, t_0 \,\text{then}\, t_1 \,\text{else}\, t_2$ with typing rules from the lecture. We extend the definition of substitution by the following three cases:

...
$[x \mapsto s]\, \text{false} = \text{false}$
$[x \mapsto s]\, \text{true} = \text{true}$
$[x \mapsto s]\, \text{if}\, t_0 \,\text{then}\, t_1 \,\text{else}\, t_2 = \text{if}[x \mapsto s]t_0 \,\text{then}[x \mapsto s]t_1 \,\text{else}[x \mapsto s]t_2$

Show that if $\Gamma, x : S \vdash t : T$ and $\Gamma \vdash s : S$ then $\Gamma \vdash [x \mapsto s]t : T$. Hint: try induction on the typing derivation $\Gamma, x : S \vdash t : T$.