In order to be admitted to the exam, you have to successfully submit your homework every week, except for 2 weeks. A successful submission is one where you get at least 1 point.

**Handin** Please submit this homework until Thursday, November 29, either via email to Philipp Schuster (philipp.schuster@uni-tuebingen.de) before 12:00, or on paper at the beginning of the lab.

**Groups** You can work in groups of up to 2 people. Please include the names and Matrikelnummern of all group members in your submission.

**Points** For each of the Tasks you get between 0 and 2 points for a total of 6 points. You get:
- 1 point, if your submission shows that you tried to solve the task.
- 2 points, if your submission is mostly correct.

### Task 1: Types

Consider the following programming language:

\[
\langle \text{term} \rangle ::= \text{\textquoteleft sqop\textquoteright} | \text{\textquoteleft squiggle\textquoteright} \langle \text{term} \rangle | \text{\textquoteleft squaggle\textquoteright} \langle \text{term} \rangle | \text{\textquoteleft transmogrify\textquoteright} \langle \text{term} \rangle \langle \text{term} \rangle
\]

We define the set of types as:

\[
\langle \text{type} \rangle ::= \text{\textquoteleft W\textquoteright}
\]

The typing relation is defined by the following rules. We adopt the convention that \( t \in \text{term} \) and \( T \in \text{type} \).

- **T-SQOP**
  - \( \text{sqop} : W \)

- **T-SQUIGGLE**
  - \( t : W \)
  - \( \text{squiggle} t : W \)

- **T-SQUAGGLE**
  - \( t : T \)
  - \( \text{squaggle} t : T \)

Which of the following terms are well-typed? Prove well-typedness by drawing a derivation tree for those terms that are well-typed. For those terms that are not well-typed it is enough to briefly note that fact.
1. squiggle sqop
2. squaggle(squiggle(squaggle sqop))
3. squaggle(transmogrify(squiggle sqop)(squaggle sqop))

**Task 2: Normal form**

Consider the language from Task 1 again. We define the set of values as:

\[
\langle \text{value} \rangle ::= \text{'sqop'} \mid \text{'squiggle'} \langle \text{value} \rangle
\]

We define an evaluation relation as:

\[
\begin{align*}
\text{E-SQUIGGLE} \quad & t \longrightarrow t' \\
\text{squiggle} \quad & t \longrightarrow \text{squiggle} t' \\
\text{E-SQUAGGLE} \quad & \text{squaggle} t \longrightarrow t
\end{align*}
\]

For each of the three terms in Task 1, answer the following four questions:

1. Let the term be \( t \). Is there a term \( t' \) such that \( t \longrightarrow t' \)? If so, draw a derivation tree.
2. Is the term in normal form? No proof required.
3. Is the term a value? No proof required.
4. Is the term stuck? No proof required.

**Task 3: Progress and Preservation**

Prove the following for the language defined in Task 1 and Task 2:

1. **Progress**: For all \( t \in \text{term} \) and \( T \in \text{type} \) such that \( t : T \) holds: either \( t \) is a value or there exists a \( t' \in \text{term} \) such that \( t \longrightarrow t' \).
2. **Preservation**: For all \( t, t' \in \text{term} \) and \( T \in \text{type} \), if \( t : T \) and \( t \longrightarrow t' \) then \( t' : T \).